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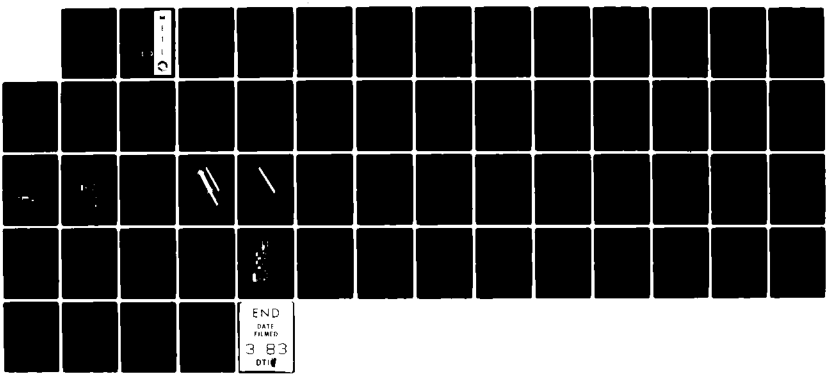

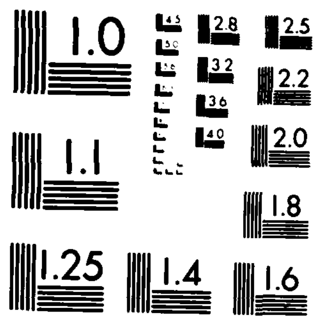
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Measurement techniques of electrical parameters of surface materials in the X-band region

Frederick W. Rohde

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OCTOBER 1982

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Based on an in-depth study of the propagation of EM plane waves and TE_{10} waves in materials, the relationships between wave propagation parameters and the electrical parameters of the materials are established. Quantities that are related to wave propagation parameters and can be measured are defined. Potential measurement techniques are analyzed and evaluated. Two measurement techniques are identified as suitable for surface material measurements. A measurement system called Radar Parameter Instrumentation is described.

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PREFACE

This study was conducted under DA Project 4A762707A855, Task CO, Work Unit 0016, "Measurement Techniques of Electrical Parameters of Surface Materials in the X-Band Region."

The study was done during the period March 1981 to March 1982 under the supervision of Mr. Melvin Crowell, Jr., Director, Research Institute.

COL Edward K. Wintz, CE was Commander and Director and Mr. Robert P. Macchia was Technical Director of the Engineer Topographic Laboratories during the report preparation.

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MEASUREMENT TECHNIQUES OF ELECTRICAL PARAMETERS OF SURFACE MATERIALS IN THE X-BAND REGION

INTRODUCTION

Radar signatures from terrain may be used for mapping, reconnaissance, or surveillance. The signatures are displayed on film or display devices such as CRTs. A radar signature is determined by the radar system, the geometry of the radar relative to the target, and the shape and reflectivity of the target. The reflectivity is a function of the electrical parameters of the target material and the roughness of the target surface. A successful interpretation of radar imagery requires a knowledge of the electrical paramaters of the targets. There are very few data available on electrical parameters of terrain surface materials, particularly for the X-band region. A research of this subject revealed that there exists no useful measurement technology in the X-band region to determine the electrical parameters of surface materials efficiently with adequate accuracy.

This research note deals with the propagation of EM waves in lossy materials, the relationships between wave propagation parameters and electrical parameters of materials, and the development and discussion of measurement techniques that are suitable for surface material measurements. Various measurement methods are integrated into a Radar Parameter Instrument System and their potentials and limits are evaluated. Some measurement results are presented.

EM WAVES AND ELECTRICAL PARAMETERS OF MATERIALS

The electrical field component of an EM wave traveling in the x-direction is given by

$$E(x) = E_o \exp(\pm \gamma x) \exp(j\omega t) \qquad (1)$$

The amplitude of E(x) is E_o. The term $\exp(\pm\gamma x)$ is called the propagation factor and $\exp(j\omega t)$ the time factor. The time factor is of little interest here and may be ignored. The propagation factor contains the propagation constant γ and the coordinate x. The magnetic field component of an EM wave traveling in the x-direction is given by

$$H(x) = H_o \exp(\pm\gamma x) \exp(j\omega t) \qquad (2)$$

The amplitude of H(x) is H_o. The ratio of E(x) to H(x) is called the wave impedance Z(x).

$$Z(x) = E(x) / H(x) \qquad (3)$$

In the following, electric field components and magnetic field components are simply referred to as electric components and magnetic components of the EM waves. The electric and magnetic components of EM waves are always perpendicular to each other. The electric components of the EM waves discussed in this report are also perpendicular to the direction of propagation. Because the electric component of the EM wave is more accessible to measurements, most of the equations of this report are written for the electric components. In the following, the wave forms will be analyzed at a fixed point in time. It is reasonable and convenient to set the fixed point in time so that the time factor $\exp(j\omega t)$ is equal to one. For this assumption equation (1) is reduced to

$$E(x) = E_o \exp(\pm\gamma x) \qquad (4)$$

The negative or positive sign in (4) indicates whether the wave propagates in the positive or negative direction, respectively, of the x-coordinate.

PLANE WAVES. The propagation constant γ for plane EM waves is given by

$$\gamma - sqr(-\omega^2 \varepsilon\varepsilon_0 \mu\mu_0) = k \tag{5}$$

The symbols in equation (5) are explained in the glossary. Using the equations given in the glossary and setting μ equal to one, equation (5) can be transformed into the simple form

$$k = k_o \sqrt{-\varepsilon} \tag{6a}$$

The relative permittivity ε for free space is equal to one. Hence, equation (6a) becomes for free space.

$$k = j\ k_o \tag{6b}$$

For lossy media the relative permittivity ε is complex and given by $\varepsilon = \varepsilon' - j\varepsilon''$. The electromagnetic energy loss which occurs when the EM wave travels through a lossy medium is represented by the relative loss factor ε''. The loss tangent $\tan\delta$ and the conductivity σ is related to ε'' as shown in the glossary. The loss parameters ε'', $\tan\delta$, and σ account for all dissipative effects and may represent an actual conductivity caused by migrating charge carriers as well as refer to an energy loss associated with a frequency dependent dispersion. The loss parameters can be related to the attenuation constant α of the medium. The losses of electromagnetic energy are converted into thermal energy or heat. This process is well known from the operation of microwave ovens.

If the losses are very small, ε'' can be neglected. The relative electric permittivity ε is then approximately equal to ε', and the propagation constant k is equal to $jk_o\sqrt{\varepsilon'}$. With this the electrical component of a plane EM wave in a nonlossy medium becomes

$$E(x) = E_o \exp(\pm jk_o \sqrt{\varepsilon'}\ x) \tag{7}$$

Interpretation of $k_o\sqrt{\varepsilon'}$ leads to the expression $\lambda_o / \sqrt{\varepsilon'}$, which is the wavelength λ in the nonlossy medium.

For lossy media the propagation constant is according to equation (6a) given by

$$k = k_o\ sqr(-\varepsilon' + j\varepsilon'') \tag{8}$$

The separation of equation (8) in its real and imaginary components yields

$$k = a + jb \tag{9}$$

where

$$a = k_o \mathrm{sqr}[-\varepsilon' + \mathrm{sqr}(\varepsilon'^2 + \varepsilon''^2)]/\sqrt{2} \tag{10a}$$

and

$$b = k_o \mathrm{sqr}[\varepsilon' + \mathrm{sqr}(\varepsilon'^2 + \varepsilon''^2)]/\sqrt{2} \tag{10b}$$

The terms a and b are called attenuation constant and phase shift constant. The attenuation constant can be measured. Equation (10b) provides for establishing the relation between the electric permittivity $\varepsilon = \varepsilon' - j\varepsilon''$ and the wavelength λ. Using b $2\pi/\lambda$, equation (10b) can be transformed into

$$\lambda = \sqrt{2}\lambda_o/\mathrm{sqr}[\varepsilon' + \mathrm{sqr}(\varepsilon'^2 + \varepsilon''^2)] \tag{11}$$

With k = a + jb the propagation of the electrical component of a plane EM wave is given according to (4) by

$$E(x) = E_o \exp(-ax) \quad \exp(\pm jbx) \tag{12}$$

The negative sign in exp(-ax) must be chosen because the wave is being attenuated. Plane waves are relatively easy to treat theoretically but offer only limited applications for measurements of electrical parameters. Because radar waves are frequently treated as plane waves, an understanding of the behavior of plane waves is required.

TE_{10} WAVES IN RECTANGULAR WAVEGUIDES. Rectangular waveguides have a simple geometric structure. In contrast to coaxial transmission lines waveguides do not require inner conductors and internal supports and can therefore easily be filled with material. Figure 1 shows a rectangular waveguide. The wave propagates in the positive x-direction. Figure 2 shows the instantaneous electric field distribution of a TE_{10} wave within the cross section of the waveguide. The advantages of using waveguides for measurements in the X-band region are the confinement of the wave field to the interior of the waveguide, the size and compactness of the waveguides, the ease of inserting material into the waveguide, setting

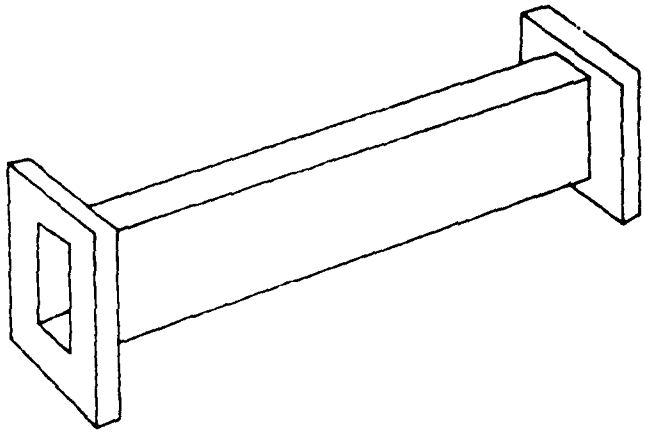

Figure 1. Rectangular Waveguide.

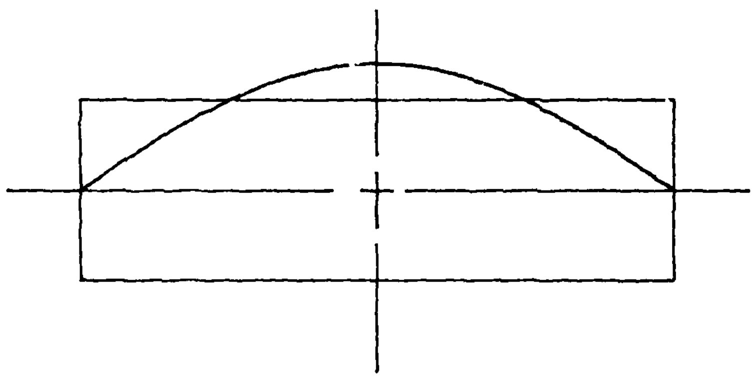

Figure 2. Electric Field Distribution of a TE_{10} Wave in the Cross Section of a Rectangular Waveguide.

the boundaries between two media into the cross section of the waveguide, and the ability to use slots in the waveguide to measure the electric field distribution.

The propagation constant λ of a TE_{10} wave traveling along the x-axis of a rectangular waveguide is given by

$$\gamma = \mathrm{sqr}[\pi^2/s^2 - \omega^2 \varepsilon\varepsilon_0 \mu\mu_0] = \Gamma \tag{13}$$

where s, a waveguide dimension, is depicted in figures 1 and 2. Using the relations as explained in the glossary, the equation (13) becomes

$$\Gamma = k_0 \mathrm{sqr}[\lambda_0^2/4s^2 - \varepsilon' + j\varepsilon''] \tag{14}$$

Separating the real and imaginary part of equation (14) gives

$$\Gamma = \alpha + j\beta \tag{15}$$

where

$$\alpha = k_0 \mathrm{sqr}\{\lambda_0^2/4s^2 - \varepsilon' + \mathrm{sqr}[(\varepsilon' - \lambda_0^2/4s^2)^2 + \varepsilon''^2]\} / \sqrt{2} \tag{16a}$$

and

$$\beta = k_0 \mathrm{sqr}\{\varepsilon' - \lambda_0^2/4s^2 + \mathrm{sqr}[(\varepsilon' - \lambda_0/4s^2)^2 + \varepsilon'^2]\} / \sqrt{2} \tag{16b}$$

The parameters α and β are the attenuation constant and phase shift constant of the TE_{10} wave in the waveguide. Attenuation constant α and phase shift constant β are always real. The sign of the phase shift constant may be positive or negative depending in what direction the wave is propagating. The wavelength of a waveguide is denoted by Λ to distinguish it from the wavelength λ of a plane wave. The wavelength of a TE_{10} wave then becomes

$$\Lambda = 2\pi/\beta = \sqrt{2}\lambda_0/\mathrm{sqr}\{\varepsilon' - \lambda_0^2/4s^2 + \mathrm{sqr}[(\varepsilon' - \lambda_0^2/4s^2)^2 + \varepsilon''^2]\} \tag{17}$$

with $\Gamma = \alpha + j\beta$ the propagation of the electrical component of a TE_{10} wave is given by

$$E(x) = E_0 \exp(-\alpha x) \exp(\pm j\beta x) \tag{18}$$

Attenuation constant α and wavelength Λ can be measured and ε' and ε'' can then be determined using the following equation

$$\varepsilon' = \lambda_o^2[1/4s^2-(\alpha^2-4\pi^2/\Lambda^2)/4\pi^2] \tag{19}$$

and

$$\varepsilon'' = \alpha\lambda_o^2/\pi\Lambda \tag{20}$$

If the interior of the waveguide is empty, the components of the relative electric permittivity are $\varepsilon' = 1$ and $\varepsilon'' = 0$ and the wavelength becomes

$$\Lambda_o = \lambda_o/\mathrm{sqr}(1-\lambda_o^2/4s^2) \tag{21a}$$

and

$$\beta_o = 2\pi/\Lambda_o \tag{21b}$$

For $\lambda_o = 2s$, Λ_o becomes infinite and no propagating wave exists. For $\lambda_o > 2s$, Λ_o becomes imaginary and a decaying field exists. The frequency $f = f_c = c/2s$ is called *the cut-off frequency of the waveguide*. If s is measured in meter [m] and f in gigahertz [GHz], the cut-off frequency becomes

$$f_c\ [\mathrm{GHz}] = 0.15/s \tag{22}$$

If the material is nonlossy, that is $\varepsilon'' = 0$, the attenuation factor α is zero and the wavelength Λ becomes

$$\Lambda = \lambda_o/\mathrm{sqr}(\varepsilon'-\lambda_o^2/4s^2) \tag{23}$$

PROPAGATION OF EM WAVES AT INTERFACES

The treatment of the propagation of EM waves at interfaces is important because the results of it can be exploited directly for the development of measurement techniques. An interface is formed when two media border each other. The interfaces discussed in this chapter are assumed to be plane and the medium to the left of the interface is assumed to be air having relative electric permittivity of $\varepsilon = 1$. The x-axis, which is the direction of wave propagation, is perpendicular to the interface and the origin of the x-axis is at the interface.

Figure 3 shows an incident wave coming from the left and traveling in the direction of the positive x-axis. The wave is split at the interface in two components. One part is reflected back into the empty half space, the other part is transmitted into the medium characterized by the electric permittivity $\varepsilon = \varepsilon' - j\varepsilon''$.

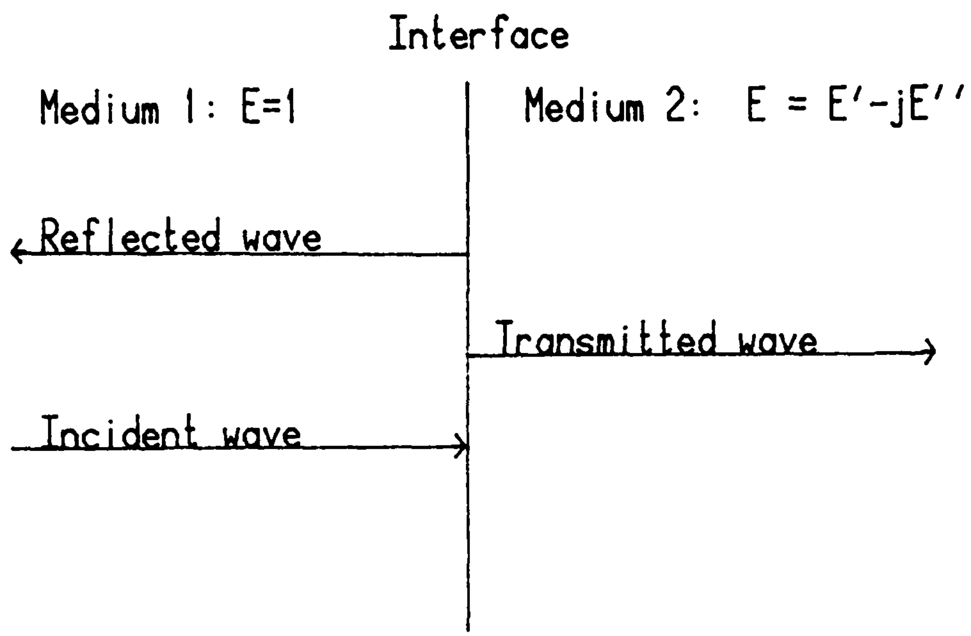

Figure 3. Incident, Reflected, and Transmitted Wave at Interface Between Two Media.

Two interface problems dealing with TE_{10} waves in waveguides are discussed in the following. In the first case the waveguide section to the right of the interface is filled with the medium having the electricity permittivity $\varepsilon = \varepsilon' - j\varepsilon''$. This section extends infinitely to the right. This case is referred to as the half space case. In the second case the waveguide section to the right of the interface is terminated by a metallic short at the distance d from the interface. This case is called the single layer case. In both cases the electric and magnetic field components and amplitudes to the left of the interface are designated by the index one, and the components to the right of the interface are designated by the index two.

HALF SPACE CASE. Figure 4 shows the arrangement of the half space case. The electric component of the field in the empty section of the waveguide (left of the interface) is the superposition of the components of the incident wave and the wave reflected at the interface. Thus

$$E_1(x) = E_{01}[\exp(-j\beta_o x) + R\exp(j\beta_o x)] \tag{24}$$

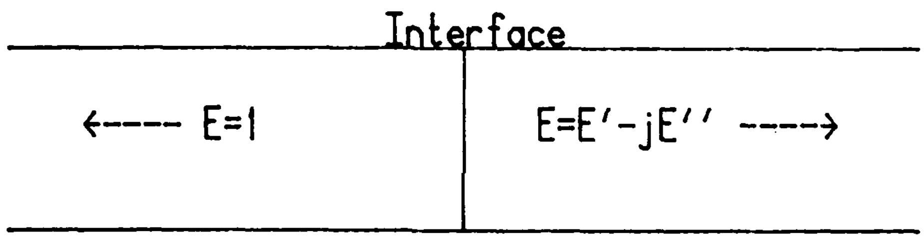

Figure 4. Half Space Problem

The symbol β_o represents the phase shift constant in the empty waveguide and is related to the wavelength Λ_o by $\beta_o = 2\pi/\Lambda_o$. The factor $R = re^{j\theta}$ is called the complex reflection coefficient of the interface.

The magnetic component to the left of the interface is given by

$$H_1(x) = H_{01}\ [\exp(-j\beta_o x) - R\exp(j\beta_o x)] \tag{25}$$

where the amplitude H_{01} is equal to $-\beta_o\ E_o/\omega\mu_o$.

The wave impedance of the empty waveguide at the interface x = 0 is given by

$$Z_1(0) = E_1(0)/H_1(0) = -\omega\mu_o(1+R)/\beta_o(1-R) \tag{26}$$

The electrical component of the EM wave in the waveguide section to the right of the interface which is filled with the medium having the electric permittivity $\varepsilon = \varepsilon' - j\varepsilon''$ is given by

$$E_2(x) = E_{02}\ \exp(-\Gamma z) \tag{27}$$

where $\Gamma = \alpha + j\beta$

The corresponding magnetic component is given by

$$H_2(x) = H_{02}\ \exp(-\Gamma z) \tag{28}$$

where the amplitude H_{02} is equal to $j\Gamma\ E_{02}/\omega\mu_o$.

The wave impedance of the waveguide filled with the material at the interface x = 0 is given by

$$Z_2(0) = E_2(0)/H_2(0) = \omega_o\mu_o/j\Gamma \tag{29}$$

The continuity of the electric and magnetic field components at the interface requires that

$$Z_1(0) = Z_2(0) \tag{30a}$$

Inserting the expressions $Z_1(0)$ and $Z_2(0)$ from (26) and (29) respectively yields

$$j/(\alpha+j\beta) = (1+R)/(1-R)\beta_o \tag{30b}$$

Using the relationship $R = re^{j\theta} = r(\cos\theta + j\ \sin\theta)$ and equation (30b), α and β can according to appendix B be expressed by

$$\alpha = 2\beta_o\ r\ \sin\theta/(1+2r\ \cos\theta+r^2) \tag{31}$$

$$\beta = \beta_o\ (1-r^2)/(1+2r\ \cos\theta+r^2) \tag{32}$$

Using equation (23) and the relation $\beta_o = 2\pi/\Lambda_o$, the phrase shift constant β_o of a TE_{10} wave can be expressed by

$$\beta_o = 2\pi\sqrt{1-(\lambda_o/2s)}\ /\lambda_o \tag{33}$$

The tree space wavelength λ_o, the waveguide dimension s, and the parameters r and θ in quotations (31) and (32) can be determined from measurements. Hence, α and β can be determined using equations (31) and (32). In equations (19) and (20) Λ can be replaced by $\Lambda = 2\pi/\beta$ which yield

$$\varepsilon' = \lambda_0^2\ [1/4s^2-(\alpha^2-\beta^2)/4\pi^2] \tag{34}$$

and

$$\varepsilon'' = \alpha\lambda_0^2\beta/2\pi^2 \tag{35}$$

Hence, the electric permittivity of the medium to the right of the interface can be determined from measurements that are executed in the empty portion of the waveguide.

SINGLE LAYER CASE. Figure 5 shows the arrangement of the single layer case. The electric and magnetic components of the EM wave in the empty *section of the waveguide are formally the same equations that have been* developed for the empty waveguide section of the half space case. Thus, the equations (24) through (26) are applied here to describe the EM wave field in the empty waveguide section.

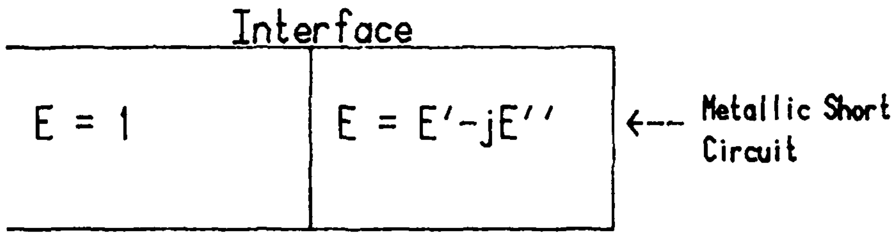

Figure 5. Single Layer Problem

The wave field in the material-filled waveguide section is the superposition of the wave that is transmitted into the medium and the wave that is reflected at the short. The electric components of the wave field in the waveguide section filled with the material having an electric permittivity $\varepsilon = \varepsilon' - j\varepsilon''$ is given by

$$E_2(x) = E_t\ \exp(-\Gamma x)+E_r\ \exp(\Gamma z) \tag{36}$$

where E_t and E_r are the amplitudes of the transmitted and reflected waves respectively at the interface. The corresponding magnetic component is given by

$$H_2(x) = \frac{1}{j\omega\mu_o} [-\Gamma \; E_t \; \exp(-j\Gamma x)+\Gamma \; E_r \; \exp(\Gamma x)] \tag{37}$$

At the short which is located at x = d the electrical component is equal to zero. Hence,

$$0 = E_t \exp(-\Gamma d)+E_r \exp(\Gamma d) \tag{38}$$

It follows from (38) that

$$E_r/E_t = -\exp(-2\Gamma d) \tag{39}$$

The wave impedance $Z_2(x)$ at the interface is

$$Z_2(0) = E_2(0)/H_2(0) = \frac{j\omega\mu_o}{\Gamma} \frac{(E_t+E_r)}{(-E_t+E_r)} \tag{40}$$

Using the relation (39) equation (40) can be transformed into

$$Z_2(0) = -(j\omega\mu_o/\Gamma) \tanh(\Gamma d) \tag{41}$$

At the interface $Z_1(0)$, given by equation (26), and $Z_2(0)$, given by equation (41), must be equal because of the continuity of the electrical and magnetic field components.

Hence, it is

$$(1+R)/(1-R) \; \beta_o = [j \tanh(\Gamma d)]/\Gamma \tag{42}$$

The left side of (42) contains quantities that can be measured, whereas the right side of (42) contains the unknown quantity $\Gamma = \alpha+j\beta$ of the material in the waveguide section to the right of the interface.

MEASUREMENT METHODOLOGY

In the preceding two chapters, relationships were established between electrical parameters of a material such as the electric permittivity, conductivity, and loss tangent and EM wave propagation parameters such as the wavelength, attenuation constant, phase shift constant, and reflection coefficient. The EM wave propagation parameter wavelength, attenuation constant, and reflection coefficient can be determined by measurements, and therefore can be used for the determination of electrical parameter of materials. The superposition of the incident and reflected wave leads to a standing wave on which measurements can be performed directly. The phase shift constant β_o in equation (24) can be replaced by the propagation constant γ of an EM wave traveling in the x-direction. It is then

$$E(x) = E_o[\exp(-\gamma x)+R\exp(\gamma x)] \qquad (43)$$

Equation (43) applies to plane interfaces, which may be formed by any two media, and is valid for TE_{10} waves as well as plane EM waves. Because E(x) is generally complex, the magnitude A(x) or E(x) is given by the square root of the product E(x) E*(x), where E*(x) is the complex conjugate of E(x).

With $\gamma = \gamma' + j\gamma''$ and $R = r\exp(j\theta)$ the magnitude A(x) becomes

$$A(x) = E_o\ \mathrm{sqr}\{[\exp(-\gamma'x-j\gamma''x)+r\exp(\gamma'x+j\gamma''x+j\theta)][\exp(-\gamma'x+j\gamma''x) + r\exp(\gamma'x-j\gamma''x-j\theta)]\} \qquad (44)$$

Using trigonometric functions, equation (44) can be transformed as follows

$$A(x) = E_o\ \mathrm{sqr}[\exp(-2\gamma'x)+r^2\exp(2\gamma'x)+ 2r\cos(2\gamma''x-\theta)] \qquad (45)$$

The propagtion constant γ for a plane wave in free space is equal to j k_o. The real component of γ is then equal to zero and γ'' becomes K_o. With γ equal to j k_o equation (45) becomes

$$A(x) = E_o\ \mathrm{sqr}[1+2r\cos(_{-}k_ox-\pi)+r^2] \qquad (46)$$

Because the cosine in equation (46) varies between +1 and -1, A(x) varies between $A_{max} = E_o(1+r)$ and $A_{min} = E_o(1-r)$.

The ratio A_{max}/A_{min} is called the Voltage Standing Wave Ratio or VSWR. It is

$$VSWR = (1+r)/(1-r) \qquad (47)$$

and

$$r = (VSWR-1)/(VSWR+1) \qquad (48)$$

The first minimum of A(x) from the interface is reached at the location of $(-x_o)$ when the argument of the cosine of (46) is equal to $\pm\pi$. It follows from this condition and the fact that $|\theta|$ cannot be larger than π that the phase angle θ becomes

$$\theta = 2k_o x_o - \pi \qquad (49)$$

To advance from one minimum to the next mimimum, or from one maximum to the next maximum, the argument of the cosine in equation (46) must be changed by 2π. It follows from this condition

$$|x_1 - x_2| = \lambda_o/2 \qquad (50)$$

where x_1 and x_2 are the locations of adjacent maxima or minima. In other words, the distance between two adjacent maxima or minima is equal to half of the wavelength.

The propagation constant of a TE_{10} wave in an empty waveguide is β_o. If k_o in (46) is replaced by β_o, the amplitude function A(x) of the standing wave becomes

$$A(x) = E_o \, sqr[1+2r \cos(2\beta_o x - \pi) + r^2] \qquad (51)$$

Expressions for VSWR, r, θ, and Λ_o can be derived for TE_{10} waves in a similar way as for plane waves, which are shown in equations (47) through (50). In fact, equations (47) and (48) are valid for plane EM waves as well as TE_{10} waves. For θ and Λ_o the following relations can be derived.

$$\theta = 2\beta_o x_o - \pi \qquad (52)$$

$$\Lambda_o/2 = |x_1 - x_2| \qquad (53)$$

The propagation constant of a TE_{10} wave in a waveguide filled with material is $\Gamma = \alpha + j\beta$. By replacing γ' with α and γ'' with β the amplitude function in the waveguide becomes, according to equation (45),

$$A(x) = E_0 \ \mathrm{sqr}[\exp(-2\alpha x) + r^2 \exp(2\alpha x) + 2r \cos(2\beta x - \theta)] \qquad (54)$$

If the waveguide is shorted at the interface x = 0 by a highly conductive metal plate, the amplitude function of the standing wave becomes

$$A(x) = E_0 \ \mathrm{sqr}[2 \cosh(2\alpha x) - 2 \cos(2\beta x)] \qquad (55)$$

An interpretation of equation (55) shows that the distance between two adjacent minima is equal to the half wavelength $\Lambda/2$. Thus

$$\Lambda/2 = |x_1 - x_2| \qquad (56)$$

If the attenuation constant α is large, maxima and minima may not exist.

The amplitude functions of a plane wave and a TE_{10} wave traveling undisturbed in a lossy medium in the x-direction are given by

$$A(x) = E_0 \exp(-ax) \qquad \text{Plane Wave} \qquad (57)$$

and

$$A(x) = E_0 \exp(-\alpha x) \qquad TE_{10} \text{ Wave} \qquad (58)$$

Figures 6 and 7 show amplitude functions of the half space and single layer case respectively for waveguides.

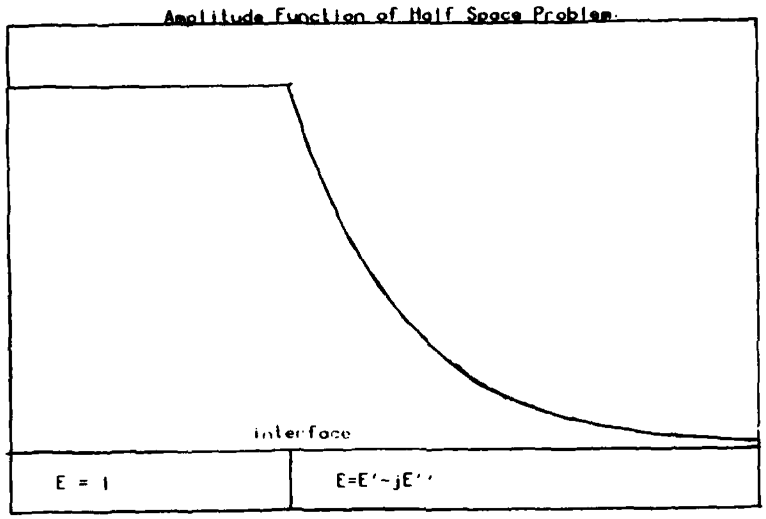

Figure 6. Amplitude Function of Half Space Problem.

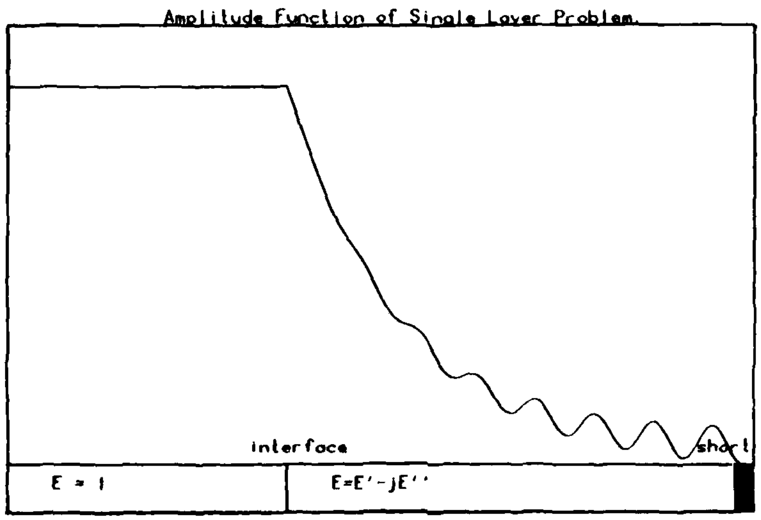

Figure 7. Amplitude Function of Single Layer Problem.

MEASUREMENT TECHNIQUES USING PLANE WAVES

Plane waves do not exist in reality but can be approximated by putting the source at a sufficiently large distance or by using antennas that produce nearly plane waves in the far field. The far field of a standard gain horn antenna consists of a wave form that is approximately a plane wave in the region of the main beam.

Figure 8 shows an arrangement for measurements of the VSWR and the distance x_0 of the first minimum from the interface. A standard gain horn antenna is mounted on a scaffold so that the antenna radiates towards the surface. The surface has to be smooth enough so that the surface scattering is not distorting the measurements. The scaffold must be designed so that it causes negligible interferences with the radiation field of the antenna. The distance of the antenna from the surface should be more than ten wavelengths. The incident wave from the antenna and the reflected wave from the surface produce a wave form that is approximately a plane standing wave. The maxima, mimina, and distance of the first minimum of the standing wave can be measured with a dipole, which is moved from the surface towards the antenna along the antenna axis. A dipole, which includes a diode for rectifying the RF voltage that is picked up by the dipole, is shown in Figure 9.

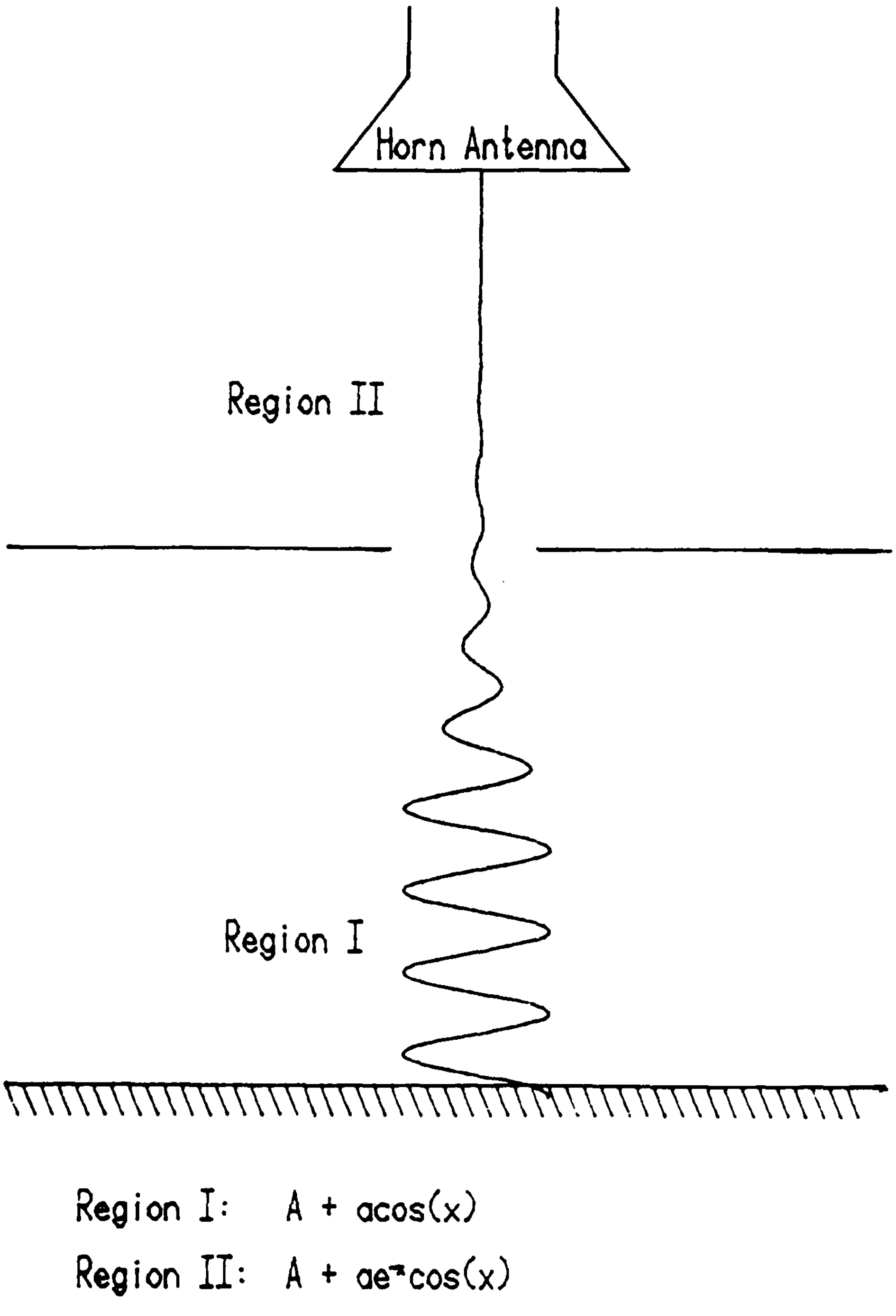

Figure 8. Plane Wave Measurement Technique.

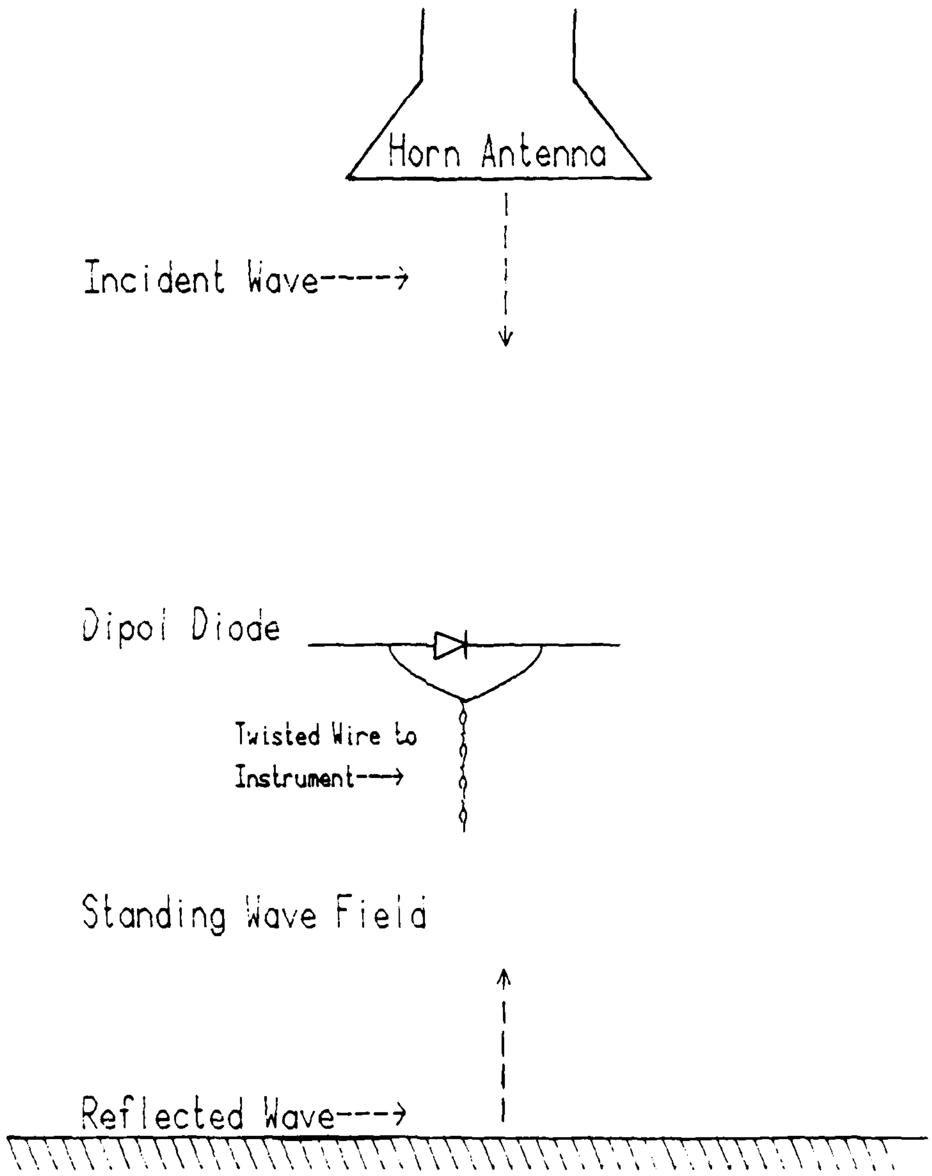

Figure 9. Dipole Probe in Standing Wave Field.

Within 3 to 5 wavelengths from the surface the VSWR is reasonably constant (within about 5 percent). Beyond that range the VSWR decreases gradually towards one at the aperature of the horn antenna. This method was successfully employed in the L-band region (1250 MHz) by one of the authors to measure electric permittivity and conductivity of surface materials in situ. The translation of this measurement technique into the X-bank encountered severe difficulties that were due to the roughly ten-times-shorter wavelength. The problem areas were the design of the dipole and the rigidity of the scaffold. Figure 10 shows the scaffold that was used for the experiments and figure 11 shows two dipole diodes that were used in the experiments. A ruler graduated in centimeters is also shown for comparison. The repeatability of the experiments was poor and the accuracy of the measurements was not better than 30 percent. To improve this measurement technique for the X-band region a significant improvement of the instrumentation in design as well as fabrication would be necessary. Because this laboratory is not equipped to perform the improved fabrication of the instrumentation, the plane wave measurement technique was no longer pursued.

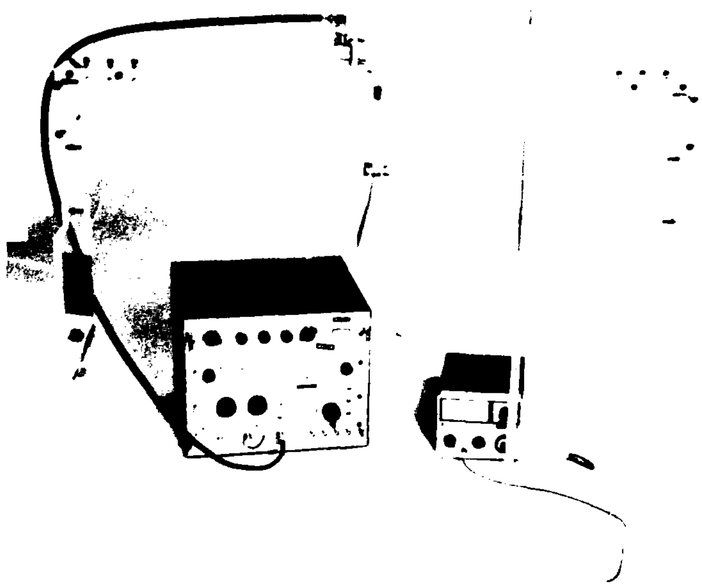

Figure 10. Radar Parameter Instrumentation for In Situ Measurements.

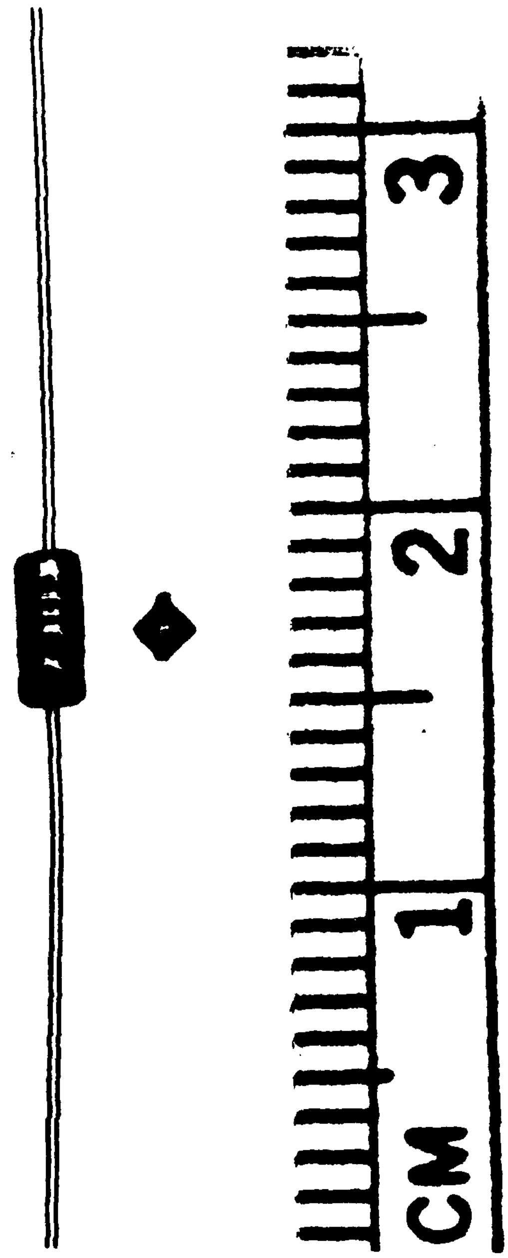

Figure 11. Dipole Diodes.

MEASUREMENT TECHNIQUES USING WAVEGUIDES

The wavelength Λ, the attenuation constant α, the VSWR, and the distance x_0 of the first minimum from the interface can be measured directly. In the following the techniques of these measurements are discussed in detail.

WAVELENGTH MEASUREMENTS. Wavelength measurements are conducted with a short-circuited slotted line. The material to be investigated is filled into a trough which fits exactly into the interior of the slotted line. Then the trough is inserted into the slotted line and the end of the slotted line is short-circuited. Figure 12 shows a trough filled with sandy soil and partially inserted into a waveguide. Figure 13 shows an empty trough. Both figures show also a ruler graduated in inches. The trough is machined from a solid piece of plexiglass. The thickness of the trough walls is less than a sixteenth of an inch. Experiments were conducted to determine the influence of the trough on the measurements. It was found that measurements with or without the trough differed by only about one percent. The short circuit causes a standing wave in the slotted line. The distance between two adjacent minima is, according to (56), equal to $\Lambda/2$. The minima of the standing wave in the slotted line can be sensed by a probe that penetrates into the slot and can be moved along the slot. The voltage which is picked up by the probe is proportional to the amplitude function of the standing wave and can be displayed by a voltmeter.

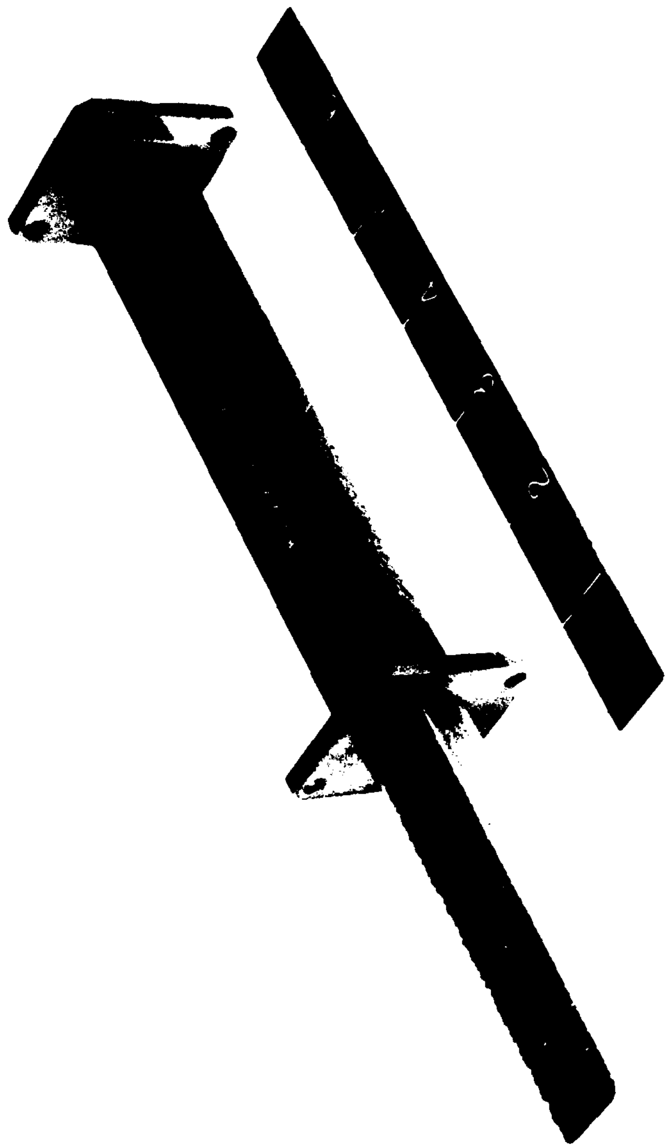

Figure 12. Plexiglass Trough Filled with Soil and Partially Inserted in Waveguide.

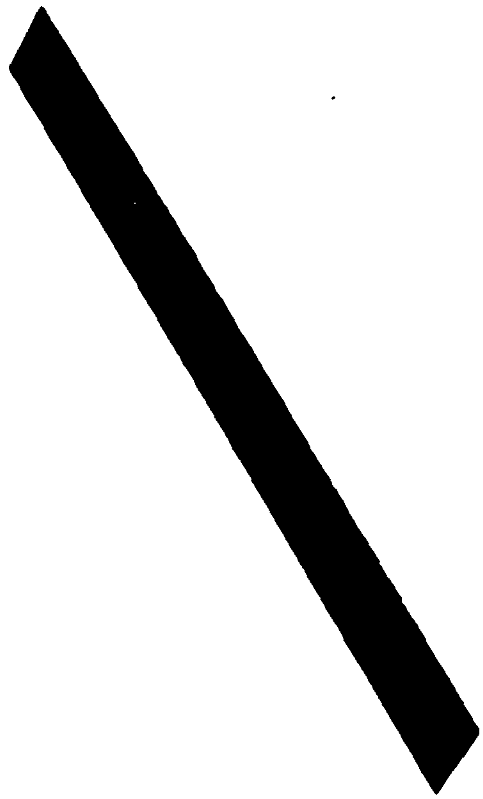

Figure 13. Empty Plexiglass Trough.

ATTENUATION CONSTANT MEASUREMENTS. Equation (58) represents the amplitude function of a TE_{10} wave that travels in a lossy material in the positive x-direction. At the locations x_1 and x_2 the amplitudes are $A(x_1)$ and $A(x_2)$ respectively. The ratio of $A(x_1)$ to $A(x_2)$ is given by

$$A(x_1)/(A(x_2) = \exp(\alpha \ell) \tag{59}$$

where ℓ is equal to $x_2 - x_1$.

From equation (59) follows that the attenuation constant α is given by

$$\alpha = \ell n[A(x_1)/A(x_2)]/\ell \tag{60}$$

The attenuation of a TE_{10} wave in a lossy medium can be measured by an experimental set-up as shown in figure 14. The EM wave coming from the generator can be routed by the two switches through either one of the two branches of waveguides to the detector. The one branch contains a calibrated attenuator and the other branch, the material under test. The material to be tested is inserted to fill a plexiglass trough to assure that the interfaces are located in cross sections of the waveguide. Both branches are adjusted first without test material by the attenuator so that both branches produce the same detector reading. After the material under test is inserted, both branches are adjusted again to the same detector reading by increasing the attenuation of the attenuator. The measured increase of attenuation in the attenuator branch is the same as the attenuation caused by the test material. The attenuation measured with the calibrated attenuator is the sum of the attenuation due to the propagation through the lossy material and the attenuation due to the reflections of the EM wave at the two interfaces that are formed by the inserted material. Because the attenuator is calibrated in power attenuation, the attenuation constant α and the magnitude r of the reflection coefficient, which are defined for voltages, have to be expressed in terms of attenuation and reflection of power. This conversion is established by the relationship $P = V^2/R$, where P is the power, V is the voltage, and R is the resistance. If the power of the incident wave at the first interface is P_o, the power of the reflected wave is equal to r^2P. The power P_1 transmitted into the lossy mediun is then

$$P_1 = P_o(1-r^2) \tag{61a}$$

or

$$r^2 = 1- P_1/P_o \tag{61b}$$

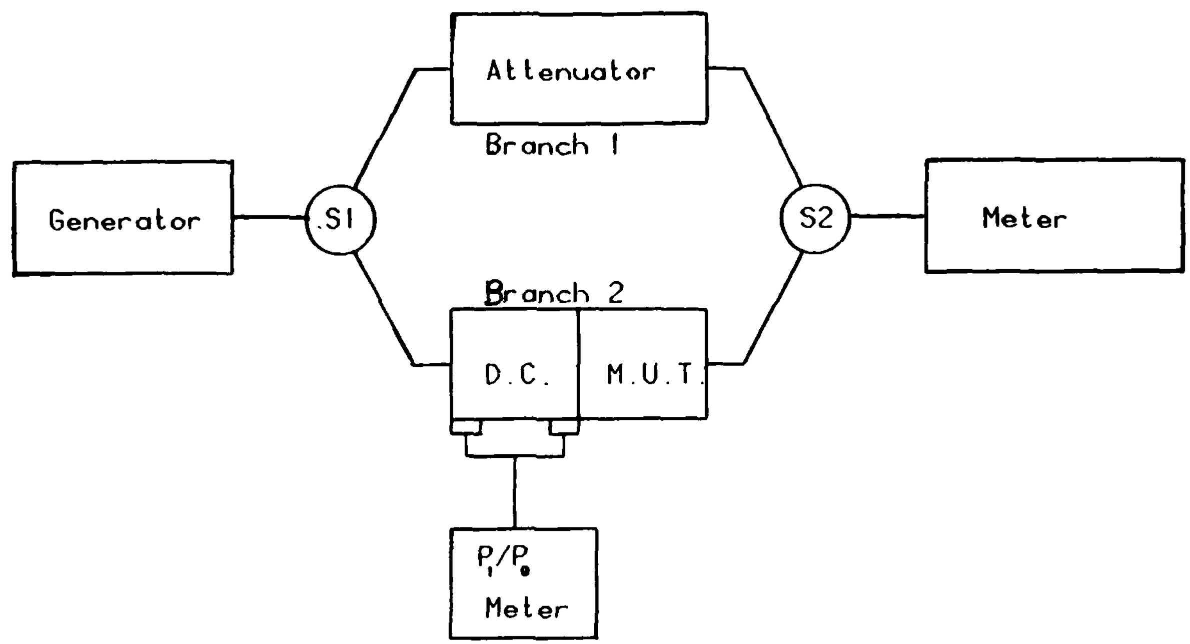

Figure 14. Attenuation Measurement Technique.

The power P_1 is attenuated by the losses in the material to the power P_2 at the second interface. It is

$$P_2 = P_1 \exp(-2\alpha) \qquad (62)$$

where ℓ is the length of the waveguide section that is filled with material. The power P_3, which is transmitted through the second interface and is measured by the detector instrument, is given by

$$P_3 = P_2(1-r^2) \qquad (63a)$$

Using (61) and (62), P_3 can be expressed as follows

$$P_3 = P_o(1-r^2)^2 \exp(-2\alpha\ell) \qquad (63b)$$

The attenuation A of the attenuator is calibrated in decibels [dB] and measures the power ratio P_3/P_o. The relationship between A and P_3/P_o is given by

$$A[dB] = 10 \log (P_3/P_o) \qquad (64)$$

Using the power ratio P_3/P_o of equation (63b) in equation (64), the attenuation becomes

$$A[dB] = 20 \log(1-r^2) \exp(-\alpha\ell) = [\log(1-r^2) - \alpha\ell/\ln 10] \qquad (65)$$

Figures 15a, 15b, and 15c show the attenuation A as a function of the length ℓ. The magnitude r of the reflection coefficient can be determined from a VSWR measurement of a slotted line that is included in the branch between the first switch and the first interface (see figure 14), or by a pair of directional couplers that are included in the branch between the first switch and the first interface. The pair of directional couplers measures the ratio of the reflected power to the incident power at the first interface, which is equal to r^2. If the reflection coefficient r is determined, the attenuation constant α can be determined from the attenuation measurement A by using equation (65). It is

$$\alpha = -\ln 10 \, [A-20 \log(1-r^2)]/20\ell \qquad (66)$$

In equations (61) through (63) multipath reflections on the interfaces have been neglected. this approximation can be made if A is at least five times larger than 20 $\log(1-r^2)$.

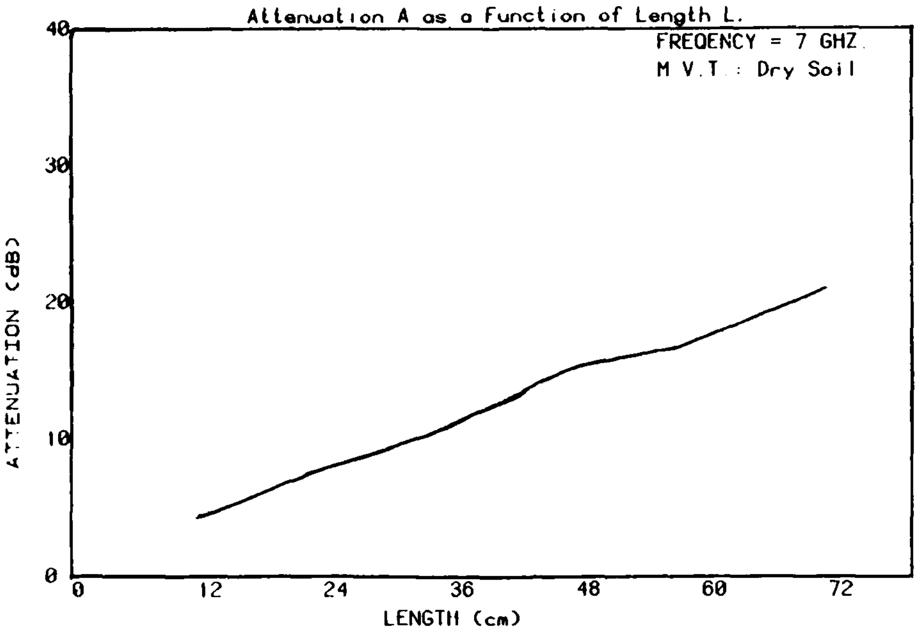

Figure 15a. Attenuation as Function of Length for Dry Soil at 7 GHz.

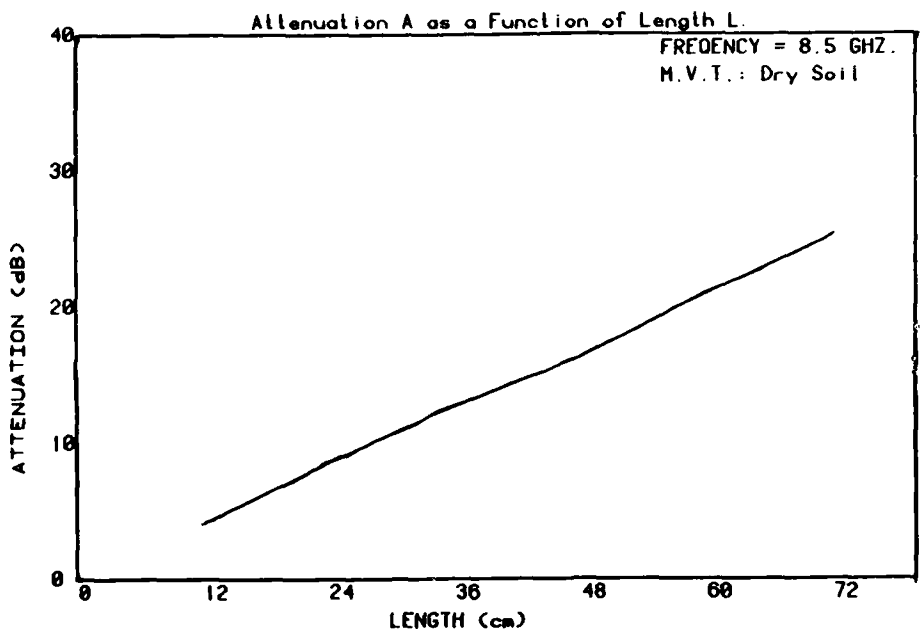

Figure 15b. Attenuation as Function of Length for Dry Soil at 8.5 GHz.

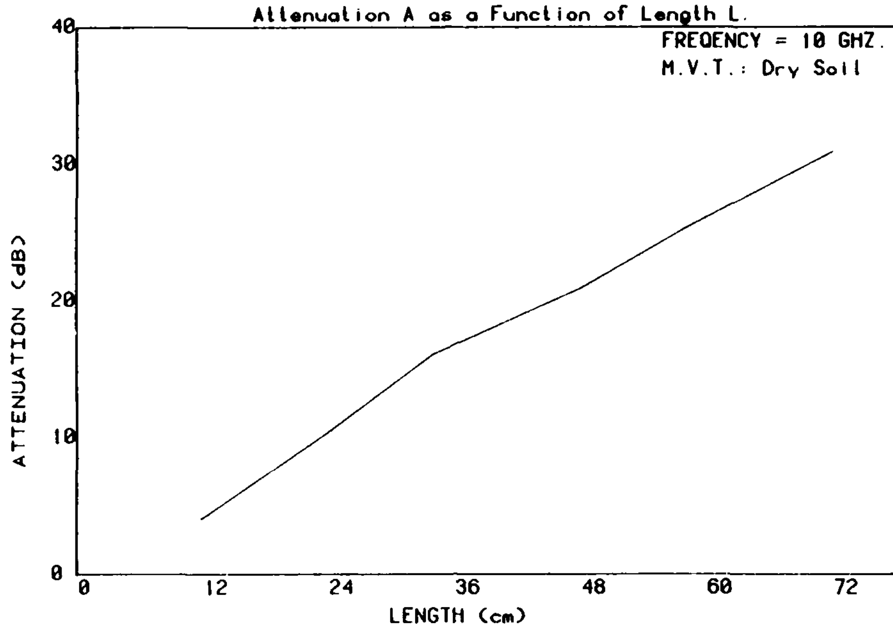

Figure 15c. Attenuation as Function of Length for Dry Soil at 10 GHz.

VSWR AND X_0 MEASUREMENTS. The voltage standing wave ratio (VSWR) and the distance x_0 of the first minimum of the standing wave from the interface are measured by using a slotted line. Figure 16 shows the experimental set-up. The EM wave is generated by the generator and transmitted to the slotted line. The end of the slotted line is first terminated by a short circuit, which produces a standing wave with sharp minima. The location ℓ_1 of a minimum close to the end of the slotted line is measured with the probe and designated as the origin of the x-axis. The short circuit is then replaced by the material under test, which is inserted into a waveguide section. If the waveguide section that is filled with the test material is terminated by a short circuit, the slotted line and waveguide section together represent the condition of the single layer case. If the waveguide section is long enough so that reflections from the end can be neglected, the slotted line and the waveguide section together represent the condition of the half space case. The termination of the slotted line with the material under test produces a standing wave with minima that are shifted in direction of the negative x-axis and less pronounced compared to the minima of the standing wave produced by the short circuit at the end of the slotted line. The location ℓ_2 of the minima that is shifted from ℓ_1 towards the generator is measured. The differences of ℓ_1 and ℓ_0 is equal to x_0. The VSWR is determined from measurements of maxima and minima of the standing wave produced by the material under test.

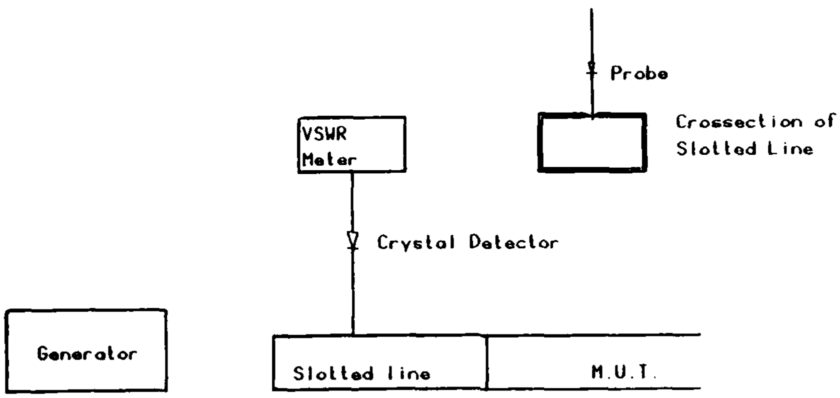

Figure 16. VSWR and X_0 Measurement Techniques.

EVALUATION OF MEASUREMENT TECHNIQUES

The measurement technique that is not constrained experimentally is based on the single layer case, which has been discussed in a previous chapter. The EM wave field in this case is entirely confined to the interior of the waveguide and no assumptions or approximations have to be made. The VSWR and x_0 measurements are performed in the empty waveguide section using a high precision slotted line. The measured quantities can be translated through a set of equations to the electrical parameters ϵ' and ϵ'' of the test material that is inserted into the short circuited waveguide section. The key of the translation process is the transcendental equation (42). It is shown in appendix B that equation (16), which corresponds to equation (42), can be solved with respect to α and β only by using a cumbersome numerical process that is complicated by ambiguities. Therefore, this method is recommended only where high accuracy is required and the material is homogeneous.

The electric permittivity of a surface material may change considerably from sample to sample. These changes are due to the relatively high variation in composition and texture that is characteristic for surface materials. Thus, it is reasonable to express the electric permittivity of a specific surface material in terms of its mean value and its standard deviation. The determination of the mean and standard deviation requires a sufficiently large number of experiments. To accomplish this the single experiment must be executed and evaluated in a relatively short time and without difficulties. The techniques for measuring VSWR and x_o based on the half space case and the techniques for measuring the wavelength Λ and attenuation A provide the means to design experiments that can be performed fast and with adequate accuracy. The half space mode of the VSWR and x_o measurements is approximated by using a waveguide section of finite rather than infinite length. A criterion for adequate length of the waveguide section that is filled with the material under test is the independence of the VSWR and x_o measurements from the termination of the waveguide. If the open-ended or short-circuited waveguide does not change the results of the VSWR and x_o measurements, the approximation is fully adequate. If there is a small change between open-ended and short-circuited termination, the open-ended termination will yield still satisfactory results because a portion of the wave energy is radiated through the open end to the outside of the waveguide. The perimeters VSWR and x_o are measured with a high-precision slotted line. The following set of equations is used for the evaluation of measurements.

$$r = (\mathrm{VSWR}-1)/(\mathrm{VSWR}+1) \tag{48}$$

$$\theta = 2\beta_o x_o - \pi \tag{52}$$

$$\beta_o = 2\pi/\Lambda_o \tag{21b}$$

$$\alpha = 2\beta_o r \sin\theta/(1+2r\cos\theta+r^2) \tag{31}$$

$$\beta = \beta_o(1-r^2)/(1+2r\cos\theta+r^2) \tag{32}$$

$$\epsilon' = \lambda_o^2\,[1/4s^2-(\alpha^2-\beta^2)/4\pi^2] \tag{34}$$

$$\epsilon'' = \alpha\lambda_o^2\beta/2\pi^2 \tag{35}$$

For practical reasons the length of the waveguide section filled with the material under test should not exceed much more than one meter. To keep the measurement accuracy at better than 5 percent the attenuation constant α of the material should not be less than 0.1 per meter.

The waveguide measurement technique using a short-circuited high-precision slotted line can be used not only to measure the wavelength Λ but also to measure the amplitude function $A(x)$ given by equation (55). The amplitude function gives a qualitative indication of the attenuation constant α because of the term $\cos h(2\alpha x)$. The evaluation of equation (55) with respect to α, however, is inconvenient. To be able to measure the distance between two adjacent minima with sufficient accuracy, the attenuation constant α of the material under test should not exceed 10 per meter. The slot of the high-precision slotted line is about 17 centimeters long. At 10 GHz seven or more minima of the standing wave may be observed. From seven or more wavelength measurements the mean and standard deviation of the wavelength can be determined adequately. Table 1 shows the result of some measurements taken with the short-circuited slotted line. If the attenuation coefficient α of the material under test is sufficiently small, e.g., not more than 0.1 per meter, the imaginary part ε'' of the electric permittivity becomes less than 1/1000. In this case α in equation (19) can be set equal to zero and ε' can be expressed by the modified equation (19) as

$$\varepsilon' = \lambda_0^2[1/4s^2+1/\Lambda^2] \tag{19a}$$

If the attenuation is not negligibly small, an attenuation measurement must be made in addition to the wavelength measurement so that ε' and ε'' can be determined. The attenuation measurement has been discussed in detail in a previous section. The power reflection coefficient r^2 equal to P_1/P_0 is determined by measuring P_0 and P_1 using the pair of directional couplers. The attenuation is measured with the attenuator in decibles. The following set of equations is used for the evaluation of the measurements.

$$\Lambda = 2\ x_1-x_2 \tag{56}$$

$$r^2 = P_1/P_0 \tag{61b}$$

$$\alpha = -\ln 10[A-20\log(1-r^2)]/20\ell \tag{66}$$

$$\varepsilon' = \lambda_0^2[1/4s^2-(\alpha^2-4\pi^2/\Lambda^2)/4\pi^2] \tag{19}$$

$$\varepsilon'' = \alpha\lambda_0^2/\pi\Lambda \tag{20}$$

Table 1. Mean Wavelength and Standard Deviations of Various Surface Materials

Material	Frequency [GHz]	Mean Wavelength Λ [cm]	Standard Deviation [cm]
Dry Fine All-Purpose Sand	10	1.97	0.20
Dry Coarse All-Purpose Sand	10	1.88	0.31
Dry Fine Top Soil I	10	2.28	0.04
Dry Fine Top Soil II	10	2.42	0.05
Dry Red Clay	10	2.47	0.04
Dry Calcium Sulfate	10	1.96	0.05
Silt	10	2.18	0.04
Wood	10	3.11	0.03
Empty Trough	10	4.00	0.02

All materials except dry coarse all-purpose sand and wood were kiln dried at a temperature of approximately 350°F for about 24 hours.

The need for determining the power reflection coefficient r^2 can be eliminated by making two or more attenuation measurements. Inserting the two lengths, ℓ_1, and ℓ_2, into equation (65) yields the equations

$$A_1 = 20 \log(1-r^2) - \alpha\ell_1/\ln 10$$

and

$$A_2 = 20 \log(1-r^2) - \alpha\ell_2)/\ln 10$$

The difference $A_1 - A_2$ is then

$$A_1 - A_2 = \alpha(\ell_2-\ell_1)/\ln 10 \qquad (67)$$

From (67) follows

$$\alpha - (A_1-A_2) \ln 10/(\ell_2-\ell_1) \qquad (68)$$

The measurement techniques involving waveguides apply only to TE_{10} waves. If the wavelength Λ is shortened because of the high electric permittivity of the material under test, different modes of propagation may occur. It was observed for certain materials at frequencies above 10 GHz that the probe of the slotted line did not pick up any measureable voltage, or that the distance between two minima was larger than expected, or that the distance changed in a somewhat periodic pattern. The observed phenomena indicate that the EM wave in the waveguide is no longer a TE_{10} type wave but is of the type TE_{mn} and/or TM_{mn}, where m and n are integers from 0 to 2. Figure 17 shows the electric field distribution of a TE_{20} wave along the side s. It can be seen that there is a minimum at the center of the waveguide, whereas the TE_{10} wave has a maximum at the center. If the frequency is decreased, the TE_{10} mode of wave propagation again becomes the dominant mode. Each material under test has to be tested first in a short-circuited slotted line to determine the frequency ranges in which the TE_{10} wave exists. This test provides also a qualitative measure of the magnitude of the attenuation. Based on the initial attenuation test, the measurement techniques for determining the electrical parameters of the material under test must be selected. Because the approach to the selection of measurement techniques depends on initial tests of the material, the measurement process is not routine, but requires decisions and selections at the various phases.

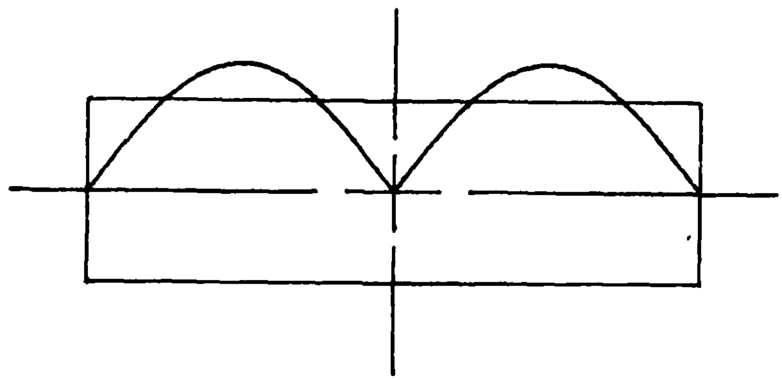

Figure 17. Electric Field Distribution of a TE_{20} Wave.

Figure 18 shows the block diagram of the Radar Parameter Instrumentation (RPI) system. This system includes all necessary instrumentation to carry out wavelength, attenuation, VSWR, and x_0 measurements. The generator, a Hewlett-Packard super high frequency (SHF) signal generator, Model 620B, provides the SHF power in the frequency range from 7 GHz to 11 GHz. Super high frequency power is modulated with a 1000 Hz square wave for detection and measuring purpose. The SHF switch S1 transmits the SHF power either to the slotted line or the attenuation measurement instrumentation. The SHF switches S2 and S3 route the SHF power through the attenuator branch or the test material branch of the attenuation measurement instrumentation. The switches S1, S2, and S3 are of the type H.B. 8761B with a frequency range from DC to 18 GHz. They are operated by a 24-volt DC power. To switch from one position to the other the polarity of the DC power has to be reversed. This is accomplished by the manually operated ON-OFF-ON switches S6, S7, and S8 which are wired in the polarity reversal mode.

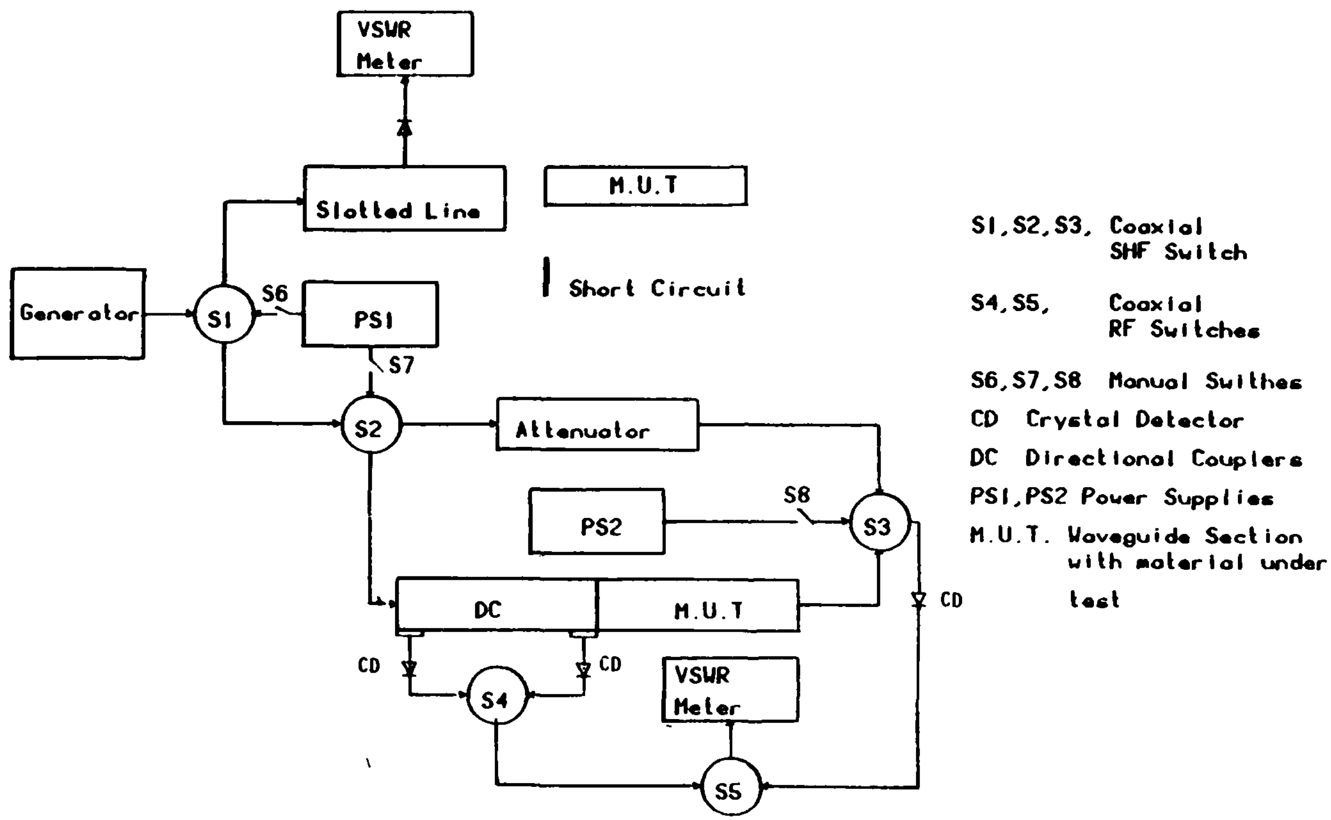

Figure 18. Block Diagram of Radar Parameter Instrumentation.

The slotted line can be used for wavelength measurements if the waveguide section of the slotted line is filled with the material under test and terminated by a short circuit. The slotted line can be used for VSWR and x_0 measurements if the slotted line is terminated by a waveguide section that is filled with the test material. The waveguide section may be open ended to approximate the half space case or terminated by a short circuit to represent the single layer case. The slotted line consists of the Slotted Waveguide Section H.P. X810B, the Slotted Line Carriage H.P. 8090, and the Detector Mount H.P. 440A. The voltage that is sensed along the slot is measured with the VSWR Meter H.P. 415E. The attenuator of the attenuation measurement instrumentation is a NARDA-704B-99 type attenuator with a frequency range from 0 to 12.4 GHz and an attenuation range from 0 to 99 dB in steps of one Db.

The pair of directional couplers consists of two Waveguide Directional Couplers H.P. X752C with a frequency range from 8.2 to 12.4 GHz and a minimum directivity of 40 dB. The incident and reflected power is demodulated by two crystal detectors of the type H.P. X424A and routed by the RF switches S4 and S5 to the VSWR Meter H.P. 415E. The H.P. 415E meter is a low-noise, tuned amplifier-voltmeter calibrated in dB and VSWR for use with square law detectors. It responds to a standard tuned frequency of 1000 Hz and has a dynamic range of 70 dB. The sensitivity is 0.15 μV rms for full-scale deflection at maximum bandwidth. The SHF power of the attenuator branch or the test material branch is demodulated by a crystal detector of the type H.P. 423A and routed through the switch S4 to the VSWR Meter H.P. 415E. The advantage of the RPI system is that it enables a change from one measurement technique to another simply by turning a few switches. Figure 19 shows a photograph of the Radar Parameter Instrumentation system.

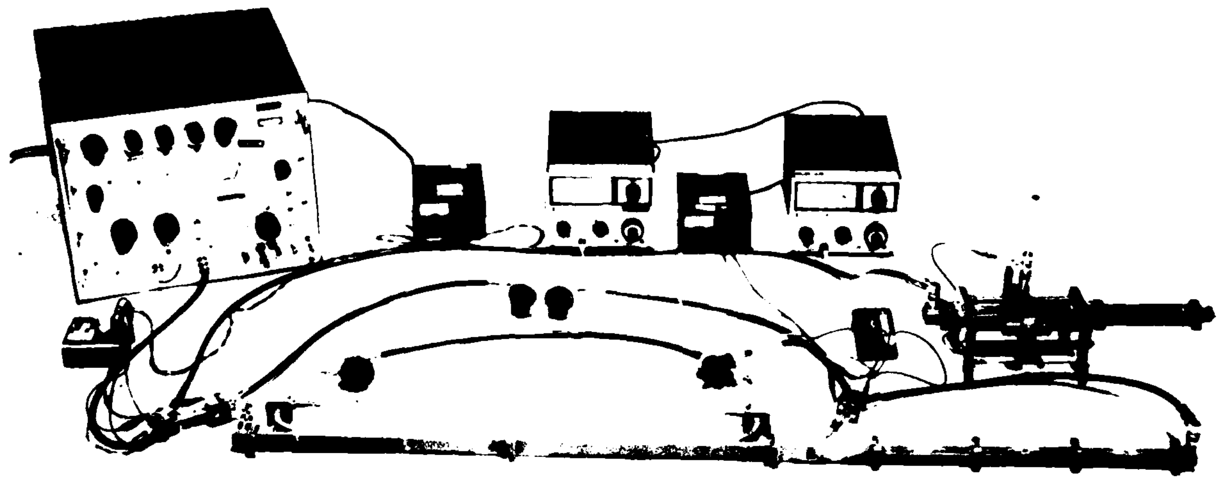

Figure 19. Radar Parameter Instrumentation for Laboratory Measurements.

CONCLUSIONS

It is concluded that

1. The plane wave method, which works successfully in the L-band region, produces unsatisfactory results in the X-band region. Possibly, a significant improvement in design and fabrication of the instrumentation would lead to acceptable results.

2. The single layer waveguide measurement technique is not suitable for determining electrical parameters of surface materials because the evaluation of the measurements presents considerable difficulties.

3. The half space waveguide measurement technique and the wavelength and attenuation measurement technique are both adequate and suitable for the determination of electric parameters of surface materials. The techniques also supplement each other. These methods are particularly convenient for investigating the influence of moisture content and texture of surface materials on their electric permittivities. The measurements can be related to the electrical parameters of the material under test by sets of relatively simple equations that can be programmed on a desk computer.

4. The selection of the measurement technique for a specific material depends on the magnitude of the electric permittivity and the loss factor of the material, and requires the development of an experimentation strategy to avoid potential mistakes.

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APPENDIX A. SEPARATION OF THE PROPAGATION CONSTANTS FOR PLANE WAVES AND TE_{10} WAVES INTO REAL AND IMAGINARY PARTS

The purpose of this appendix is to develop the appropriate expressions for the attenuation and phase constants for plane waves and TE_{10} waves. A plane wave propagating in a lossy medium in the positive x-direction will have the form $E_o \exp(-jkx)$. The quantity k^2 can be obtained from Maxwell's equations

$$k^2 = j\omega\mu_o\sigma + \omega^2\mu_o\varepsilon_o\varepsilon' = K_o^2[\varepsilon' - j\varepsilon''] \quad \text{(A1)}$$

where $\varepsilon'' = \sigma/\omega\varepsilon$.

Letting k be equal to b-ja and setting the square of it equal to equation (A1) will yield

$$k_o^2(\varepsilon' - j\varepsilon'') = b^2 - a^2 - 2jba \quad \text{(A2)}$$

Taking the real and imaginary parts of equation (A2) will give the following two equations.

$$b^2 - a^2 = k_o^2\varepsilon' \quad \text{(A3)}$$

$$2ba - k_o^2\varepsilon'' \quad \text{(A4)}$$

When equations (A3) and (A4) are solved for a and b in terms of ε' and ε'', the resulting expressions are given below

$$a = \frac{k_o}{\sqrt{2}} \text{sqr}\{\text{sqr}[\varepsilon'^2 + \varepsilon''^2] - \varepsilon'\} \quad \text{(A5)}$$

$$b = \frac{k_o}{\sqrt{2}} \text{sqr}\{\text{sqr}[\varepsilon'^2 + \varepsilon''^2] + \varepsilon'\} \quad \text{(A6)}$$

In equations (A5) and (A6) a represents the attenuation constant of the wave while b represents the phase constant.

A TE_{10} wave propagation in a rectangular waveguide filled with a lossy dielectric material will have the form $E_o \exp(-\lambda x)$. A basic expression for λ can be obtained from Maxwell's equations.

$$\lambda = \text{sqr}\{(\pi/s)^2 - \omega^2\mu_o\varepsilon_o\varepsilon' + j\omega\mu_o\sigma\} \quad \text{(A7)}$$

Setting $\rho \exp(j\phi)$ equal to the square of (A7) will give

$$(\pi/s)^2 - \omega^2\mu_o\varepsilon_o\varepsilon' + j\omega\mu_o\sigma = \rho \exp(j\phi) = \rho(\cos\phi + j \sin\phi) \qquad \text{(A8)}$$

Using (A8) and solving for ρ and ϕ will yeids the following results

$$\rho = \mathrm{sqr}\{\omega^2\mu_o^2\sigma^2 + [\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2]^2\} \qquad \text{(A9)}$$

$$\phi = \tan^{-1}\left[\frac{\omega\mu_o\sigma}{(\pi/s)^2 - \omega^2\mu_o\varepsilon_o\varepsilon'}\right] \qquad \text{(A10)}$$

An expression for λ can now be written as

$$\lambda = \rho^{1/2}\exp(j\phi/2) = [\omega^2\mu_o^2\sigma^2 + (\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2)^2]^{1/4}[\cos(\phi/2) + j \sin(\phi/2)]$$

$$\cos(\phi/2) = \mathrm{sqr}\left[\frac{1+\cos\phi}{2}\right] \qquad \sin(\phi/2) = \mathrm{sqr}\left[\frac{1-\cos\phi}{2}\right]$$

$$\cos\phi = \frac{(\pi/s)^2 - \omega^2\mu_o\varepsilon_o\varepsilon'}{\mathrm{sqr}\{\omega^2\mu_o^2\sigma^2 + [\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2]^2\}} \qquad \text{(A11)}$$

Letting $\xi = \omega^2\mu_o^2\sigma^2 + (\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2)^2$ and $\ell = (\pi/s)^2 - \omega^2\mu_o\varepsilon_o\varepsilon'$, will give the following result for λ

$$\lambda = \frac{1}{\sqrt{2}}\mathrm{sqr}[\mathrm{sqr}(\xi) + \ell] + \frac{1}{\sqrt{2}}\mathrm{sqr}[\mathrm{sqr}(\xi) - \ell] \qquad \text{(A12)}$$

or

$$\lambda = \frac{1}{\sqrt{2}}\mathrm{sqr}\{\mathrm{sqr}[\omega^2\mu_o^2\sigma^2 + (\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2)^2] + (\pi/s)^2 - \omega^2\mu_o\varepsilon_o\varepsilon'\}$$

$$+ \frac{1}{\sqrt{2}}\mathrm{sqr}\{\mathrm{sqr}[\omega^2\mu_o^2\sigma^2 + (\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2)^2] - (\pi/s)^2 + \omega^2\mu_o\varepsilon_o\varepsilon'\} = \alpha + j\beta \qquad \text{(A13)}$$

The attenuation constant (α) and the phase constant (β) can be written as

$$\alpha = \frac{1}{\sqrt{2}}\mathrm{sqr}\{\mathrm{sqr}[\omega^2\mu_o^2\sigma^2 + (\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2)^2] + (\pi/s)^2 - \omega^2\mu_o\varepsilon_o\varepsilon'\} \qquad \text{(A14)}$$

$$\beta = \frac{1}{\sqrt{2}}\mathrm{sqr}\{\mathrm{sqr}[\omega^2\mu_o^2\sigma^2 + (\omega^2\mu_o\varepsilon_o\varepsilon' - (\pi/s)^2)^2] - (\pi/s)^2 + \omega^2\mu_o\varepsilon_o\varepsilon'\} \qquad \text{(A15)}$$

Using the fact that $k_o = \omega\sqrt{\mu_o\varepsilon_o} = 2\pi/\lambda_o$ and that $\varepsilon'' = \sigma/\omega\varepsilon_o$, equations (A14) and (A15) can be rewritten as

$$\alpha = \frac{k_o}{\sqrt{2}} \mathrm{sqr}\{\mathrm{sqr}[\varepsilon''^2+(\varepsilon'-\lambda_o^2/4s^2)^2]+\lambda_o^2/4s^2-\varepsilon'\} \tag{A16}$$

$$\beta = \frac{k_o}{\sqrt{2}} \mathrm{sqr}\{\mathrm{sqr}[\varepsilon''^2+(\varepsilon'-\lambda_o^2/4s^2)^2]-\lambda_o^2/4s^2+\varepsilon'\} \tag{A17}$$

APPENDIX B. DETAILED DERIVATIONS OF THE WAVEGUIDE EQUATIONS

This appendix will present the derivation of the equations for the problem of a rectangular waveguide that is partially filled with a lossy dielectric material. Two particular cases will be considered. In the first case the lossy dielectric material will be assumed to have infinite length so that a half space problem is formulated in which no reflections from the end of the waveguide will occur. The second case will consist of a waveguide terminated in a short circuit. The geometry for the half space problem is shown in figure B1. For each case a solution will be sought for the dielectric constant and conductivity of the lossy medium in terms of the reflecton coefficient.

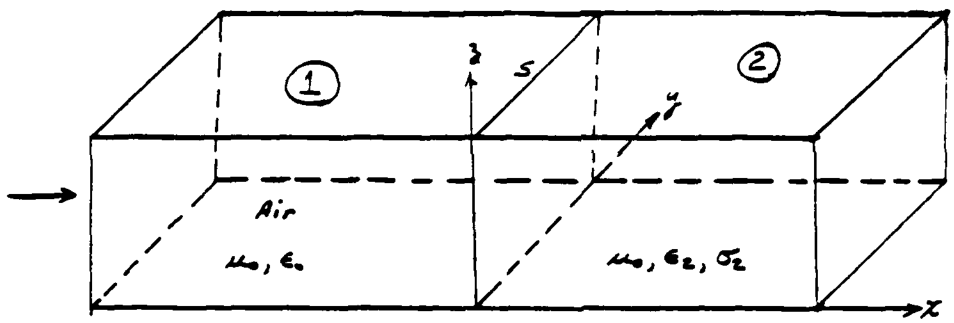

Figure B1. Half Space Geometry.

A TE_{10} wave propagates in the plus x direction and encounters the boundary at x = 0 between air and the lossy dielectric. The equations for the electric and magnetic fields in air can be written as

$$E_{1_z}(x) = E_{01}[\exp(-j\,\beta_0 x) + R\exp(j\,\beta_0 x)] \qquad \text{(B1)}$$

$$H_{1_y}(x) = (-\beta_0 E_{01}/\omega\mu_0)\,[\exp(-j\beta_0 x)] \qquad \text{(B2)}$$

where $\beta_0 = (\omega^2\mu_0\epsilon_0 - \pi^2/s^2)^{1/2}$ and R is the reflection coefficient equal to $r\exp(j\theta)$. The first subscript on the field quantities indicates the

medium in which the wave is propagating, and the second subscript indicates the direction of the field. The equations for the electric and magnetic fields in medium 2 are

$$E_2 z(x) = E_{o2} \exp(-\Gamma x) \qquad \text{(B3)}$$

$$H_{2_y}(x) = (j\Gamma E_{o2}/\omega\mu_o) \exp(-\Gamma x) \qquad \text{(B4)}$$

where $\Gamma - \alpha = j\beta$

The impedance in medium 2 (Z_2) is simply the ratio of the electric and magnetic fields.

$$Z_2 = \omega\mu_o/j(\alpha+j\beta) \qquad \text{(B5)}$$

At the boundary ($x = 0$) between air and the lossy dielectric, the wave impedance in air must be equal to Z_2.

$$E_{1_z}(0)/H_{1_y}(0) = Z_2 \qquad \text{(B6)}$$

When equations (B1) and (B2) are used on (B6) the following result is obtained

$$-(1+R)/[\beta_o(1-R)] = 1/[j(\alpha+j\beta)] \qquad \text{(B7)}$$

Separating (B7) onto real and imaginary parts will allow expressions for β and α to be obtained.

$$\beta = \beta_o\ (1-r^2)/(1+2r\cos\theta\, r^2) \qquad \text{(B8)}$$

$$\alpha = 2\ \beta_o\ r\sin\theta/(1+2r\cos\theta+r^2) \qquad \text{(B9)}$$

The quantities β and α can also be related to the medium parameters ε_2, and σ_2 in terms of r and θ.

$$\varepsilon' = 1/k_o^2\{\pi^2/s^2+[\beta_o^2(1-r^2)^2 - 4\beta_o^2 r^2\sin^2\theta]/(1+2r\cos\theta+r^2)^2\} \qquad \text{(B10)}$$

where $\varepsilon_2 = \varepsilon'\varepsilon_o$

$$\sigma_2 = (4/\omega\mu_o)[\beta_o^2 r(1-r^2)\ \sin\theta]/(1+2r\cos\theta+r^2)^2 \qquad \text{(B11)}$$

Thus we see that the dielectric constant and the conductivity of medium 2 can be determined if r and θ can be measured. The quantities r and θ can be obtained by measuring the voltage standing wave ratio and the distance to the first minimum in medium 1.

The geometry for the problem of the waveguide which is terminated on a short circuit is shown in figure B2.

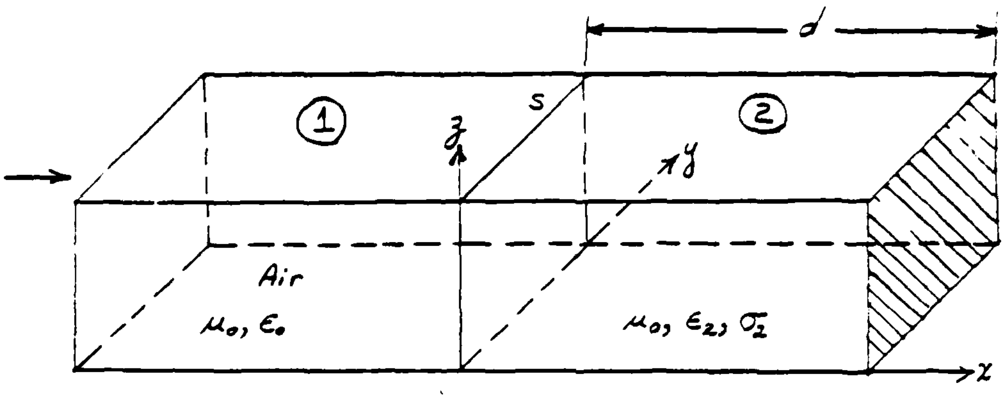

Figure B2. Short Circuit Geometry.

The distance from the plane x = 0 to the short circuit is d. The form of the electric and magnetic fields in air remains the same as those given by equations (B1) and (B2). In medium 2 the electric and magnetic fields will now have the following form.

$$E_{2_z}(x) = E_t \exp(-\Gamma x) + E_r \exp(\Gamma x) \quad o \leq x \leq d \tag{B12}$$

$$H_{2_y}(x) = -(\Gamma/j\omega\mu_o)[E_t \exp(-\Gamma x) - E_r \exp(\Gamma x)] \quad o \leq x \leq d \tag{B13}$$

At the boundary of the short circuit (x = d) we must have $E_{2_z}(d) = 0$. This boundary condition leads to the following relationship between E_t and E_r.

$$E_r = -E_t \exp(-2\Gamma d) \tag{B14}$$

The boundary conditions at x = 0 can be stated as

$$E_{1_z}(0)/H_{1_y}(0) = E_{2_z}(0)/H_{2_y}(0) \tag{B15}$$

When equations (B1), (B2), (B12), (B13), and (B14) are used in (B15), the following result for Γ can be obtained.

$$\Gamma = [j\beta_o(1-R)\tanh(\Gamma d)]/(1+R) \qquad \text{(B16)}$$

Attempts were made to solve equation (B16) for Γ using the Newton-Raphson method. This solution provides for the $n + 1^{st}$ approximation of Γ as follows.

$$\Gamma_{n+1} = \Gamma_n - f(\Gamma_n)/f'(\Gamma_n) \qquad \text{(B17)}$$

Where Γ_n represent the n^{th} approximation of Γ, and $f(\Gamma_n)$ and $f'(\Gamma_n)$ are given below.

$$f(\Gamma_n) = \Gamma_n - [j\beta_o(1-R)\tanh(\Gamma_n d)]/(1+R) \qquad \text{(B18)}$$

$$f'(\Gamma_n) = 1 - [j\beta_o d(1-R)\operatorname{sech}^2(\Gamma_n d)]/(1+R) \qquad \text{(B19)}$$

If a solution could be obtained for Γ using (B17), then the dielectric constant and the conductivity of the lossy medium could be obtained in terms of α and β, as was the case for the half space problem. However, it was found that even though (B17) converges to a solution, it is not the correct one. Equation (B16) therefore does not have a unique solution, and this was found to be the case even when two different values of d are used.

GLOSSARY

The subject of this report contains a large number of physical quantities that tend to be easily confused unless a strict nomenclature is adhered to. In this section symbols, terms, and dimensions that are used frequently throughout the report are explained. The symbols in brackets represent the units of the physical quantities. The concept of "medium" is used in the report to describe materials as well as empty space.

(x)	Electric field components of a EM wave that travels in the direction of the x-axis. E(x) represents an electrical field strength, which is measured in volts per meter [V/m]. The unit [V/m], however, is seldom used in microwave measurements. Probes which are inserted into the EM wave field are normally used for microwave measurements. They measure the relative amplitude of the electrical component of the EM wave in volts [V].
H(x)	Magnetic field component of an EM wave traveling in the direction of the x-axis and perpendicular to E(x). It is measured in ampers per meter [A/m].
x	Coordinate of propagation of the EM wave. The distance of x from the origin is measured in meters [m].
E_o	Amplitude of the electrical component of the EM wave.
H_o	Amplitude of the magnetic component of the EM wave.
t	Time. The time is measured in seconds [s].
f	Frequency. The frequency is measured in Hertz [Hz], Megahertz [MHz] = 10^6[Hz], or Gigahertz [GHz] = 10^9 [Hz]. One Hertz can also be expressed in terms of one period or cycle per second and measured in [s^{-1}].
ω	Angular frequency. The angular frequency is equal to $2\pi f$.
γ	Propagation constant of an EM wave.
$\varepsilon\varepsilon_o$	Electric permittivity of a meduim.
ε_o	Electric permittivity of free space measured in seconds per ohm·meter [s/Ω m]. The value of ε_o is $10^{-9}/36\pi$ [sΩ m].

$\varepsilon = \varepsilon' - j\varepsilon''$	Complex relative electric permittivity. It is a dimensionless number. For free space ε is equal to one.
ε'	Real part of ε.
ε''	Imaginary part of ε and called the relative loss factor. If ε'' is equal to zero, the medium having ε equal to ε' is free of electrical losses. In other words, the EM wave is not attenuated in the medium. If ε'' is larger than zero, the EM wave is attenuated and the medium is called lossy.
$\tan\delta$	Loss tangent of a medium. The loss tangent is equal to $\varepsilon''/\varepsilon'$.
σ	Conductivity of a medium. The conductivity is equal to $\omega\varepsilon_0\varepsilon''$. If the frequency is measured in Gigahertz [GHz], σ can be expressed by the equation $\sigma = \varepsilon'' f/18 [1/\Omega\ m]$.
$\mu\mu_o$	Magnetic permeability of a medium.
μ_o	Magnetic permeability of free space measured in ohm$\cdot$seconds per meter [Ωs/m]. The value of μ_o is equal to $4\pi \times 10^{-7} [\Omega\ s/m]$.
μ	Relative magnetic permeability. It is a dimensionless number and has the value of one for most media except ferromagnetic materials.
c	Propagation velocity of EM waves in free space or speed of light. It is equal to $1/\sqrt{\varepsilon_0\mu_0}$ having the magnitude of $3 \times 10^8 [m/s]$.
λ_o	Wavelength of an EM wave in free space. The product of $f\lambda_o$ is equal to the speed of light. The wavelength is measured in meters [m].
λ	Wavelength of a plane EM wave in a medium having the relative electric permittivity ε.
Λ_o	Wavelength of an EM wave in an empty (vacuum) waveguide. Because the wavelengths of an empty waveguide and an airfilled waveguide differ only by a few parts per thousand, empty and airfilled waveguides are considered to be the same in this report.

Λ — Wavelength of an EM wave in a waveguide whose interior is filled with a medium having the relative electric permittivity ε.

k — Propagation constant of a plane EM wave. Depending on the medium, k may be complex and be expressed by the relation $k = a + jb$.

k_o — Propagation constant of free space. It can be expressed in terms of λ_o by the relation $k_o = 2\pi/\lambda_o$.

a — Attenuation constant for plane EM waves. It is measured in attenuation per meter $[m^{-1}]$.

b — Phase shift constant for plane EM waves. It can be expressed in terms of λ by the relation $b = 2\pi/\lambda$.

Γ — Propagation constant of a TE_{10} wave. Depending on the medium with which the waveguide is filled, Γ may be complex and be expressed by the relation $\Gamma = \alpha + j\beta$.

β_o — Propagation constant of a TE_{10} wave in an empty waveguide. It can be expressed in terms of Λ_o by the relation $\beta_o = 2\pi/\Lambda_o$.

α — Attenuation constant for TE_{10} waves.

β — Phase shift constant for TE_{10} waves. It can be expressed in terms of Λ by the relation $\beta = 2\pi/\Lambda$.

$Z(x)$ — Wave impedance. The wave impedance is the ratio of the electrical field to the magnetic field of the EM wave and is expressed by the relation $Z(x) = E(x)/H(x)$. It is measured in ohms $[\Omega]$. The wave impedance Z_o of a plane wave in free space is equal to $377[\Omega]$. At the boundary or interface between two media the wave impedance in both media must by equal to satisfy the requirement of the continuity for the tangential electric and magnetic field components of the interface.

s — Longer side of the cross section of a rectangular waveguide. It is measured in meters [m]. In the literature the longer side of the cross section is frequently referred to by the letter a. Because the letter a is being used for the attenuation constant of a plane EM wave, the letter s is used for the length of the waveguide side.

d,l	Lengths of waveguide sections measured in meters [m].
exp(x)	Exponential function of x.
sqr(x)	Square root of x.
R	Complex reflection coefficient at an interface. R is equal to r exp(jθ).
r	Magnitude of R. The magnitude is a dimensionless number.
θ	Phase angle of R measured in radians.
VSWR	Voltage standing wave ratio. VSWR is the ratio of the maximum voltage to the minimum voltage of a standing wave.
x_o	Distance of the first minimum of a standing wave from the interface. It is measured in meters [m].
A	Attenuation measured in decibels [dB].

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